1 MT1 Homework Problem Solutions: 6

1) Simplify the following, expressing the complex numbers in the form $a+bi$:

i) $(5+22i)-(-3-1i) = 8+23i$,

[1 mark(s)]

ii) $(5+22i)\times(-3-1i) = 7-71i$,

[1 mark(s)]

iii) $\frac{5+22i}{-3-1i} = -3.7-6.1i$,

[1 mark(s)]

iv) $e^{i\pi/2} = i$.

[1 mark(s)]

2) Simplify the following, expressing the complex numbers in the form $re^{i\theta}$:

i) $(1+2i)-(-2+1i) = \sqrt{10}e^{i\theta}$, where $\theta = \arctan(1/3) = 18.4°$.

[1 mark(s)]

ii) $(1+2i)\times(-2+1i) = 5e^{i\theta}$, where $\theta = \arctan(3/4) = 216.86°$.

[1 mark(s)]

iii) $\frac{1+2i}{-2+1i} = -i = e^{2\pi i/3}$,

[1 mark(s)]

iv) $5\cos(2\pi) + 5i\sin(2\pi) = 5e^{2\pi i}$.

[1 mark(s)]

3)

i) If

$$
\begin{aligned}
z &= \frac{1}{11i} + \frac{1}{2-1i}, \\
&= \frac{-i}{11} + \frac{2+1i}{5}, \\
&= \frac{2}{5} + \frac{6i}{55}.
\end{aligned}
$$

[2 mark(s)]

ii) For $z = 1+1i$, determine z^3, $\sqrt{z}$, and zz^*

$$
\begin{aligned}
z^3 &= (1+1i)(1+1i)(1+1i) = \left[\sqrt{2}e^{i\pi/4}\right]^3 = \sqrt{8}e^{3i\pi/4} = 2i-2. \\
\sqrt{z} &= \sqrt[4]{2}e^{i\pi/8},\ \sqrt[4]{2}e^{9i\pi/8}, \\
zz^* &= (1+1i)(1-1i) = 2.
\end{aligned}
$$

[3 mark(s)]

iii) Find expressions for $\cos\theta$ and $\sin\theta$ in terms of $e^{i\theta}$.

$$\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2},$$
$$\sin\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i},$$

[2 mark(s)]

4) Write the following complex number in the form $a+bi$:

$$\begin{aligned} z &= \frac{(M_{11}-M_{22})-(i/2)(\Gamma_{11}-\Gamma_{22})}{\Delta m-(i/2)\Delta\Gamma}, \\ &= \frac{(M_{11}-M_{22})-(i/2)(\Gamma_{11}-\Gamma_{22})}{\Delta m-(i/2)\Delta\Gamma} \times \frac{\Delta m+(i/2)\Delta\Gamma}{\Delta m+(i/2)\Delta\Gamma}, \\ &= \frac{4}{4\Delta m^2+\Delta\Gamma^2}\left[(M_{11}-M_{22})\Delta m+\frac{\Delta\Gamma}{4}(\Gamma_{11}-\Gamma_{22})\right. \\ &\quad \left. +i\left(\frac{\Delta\Gamma}{2}(M_{11}-M_{22})-\frac{\Delta m}{2}(\Gamma_{11}-\Gamma_{22})\right)\right]. \end{aligned}$$

where M_{ij}, Γ_{ij}, Δm, and $\Delta\Gamma$ are constants, and $i,j=1,2$.

[3 mark(s)]

5) Find the square roots of $z=3$ and draw these on an Argand diagram (Argand diagram shown in Fig. **??**)

$$\begin{aligned} z_1 &= \sqrt{3} \\ z_2 &= \sqrt{3}e^{i\pi} \end{aligned}$$

[3 mark(s)]

1 mark for each of the roots, and the argand diagram

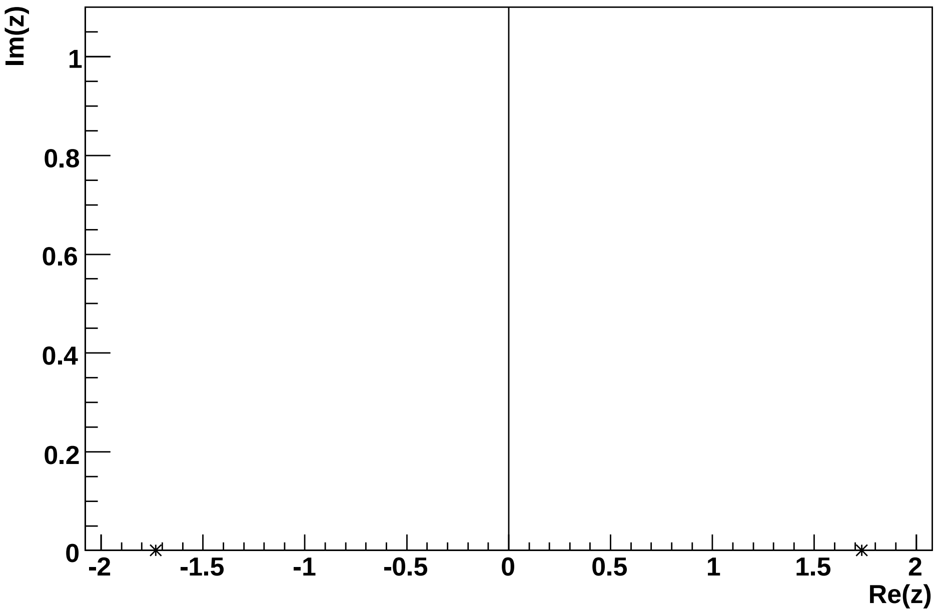

Figure 1: Square roots of $z = 3$.

6) Find the fourth roots of $z = 1$ and draw these on an Argand diagram (Argand diagram shown in Fig. **??**).

$$\begin{aligned} z_1 &= 1 \\ z_2 &= e^{i\pi/2} \\ z_3 &= e^{i\pi} \\ z_4 &= e^{3i\pi/4} \end{aligned} \tag{1}$$

[5 mark(s)]

1 mark for each of the roots, and the argand diagram

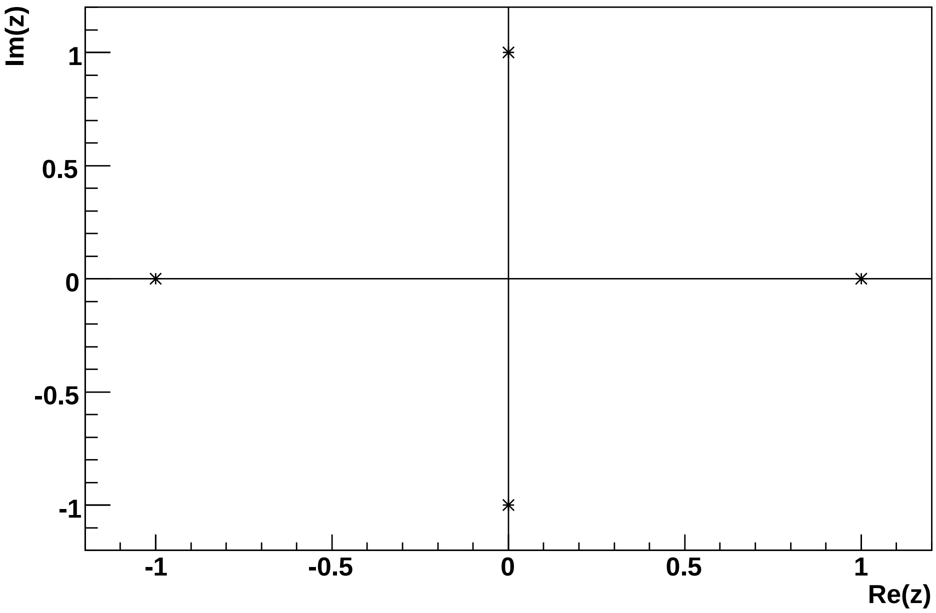

Figure 2: Fourth roots of $z = 1$.

6) Determine the Maclaurin series expansion for $f(x) = \cosh(x)$.

$$\begin{aligned} f_0 &= 1 \\ f'_0 &= 0 \\ f''_0 &= 1 \\ f'''_0 &= 0 \\ f''''_0 &= 1 \\ f'''''_0 &= 0 \end{aligned}$$

So the Maclaurin series expansion for $f(x)$ is

$$f(x) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots$$

[3 mark(s)]