PHY 241 Planetary Systems - Coursework #9

Due: Tuesday, December 14, 2010 4pm

References:

- Ch. 21 An Introduction to Modern Astrophysics 2nd Edition Astronomy, by Carroll and Ostlie (several copies in the Library)
- Ch. 13 Planetary Sciences, DePater & Lissauer (copy in the library -scan on the course website). for exoplanets

Useful quantities:

- Solar luminosity $L_\odot = 3.8 \times 10^{26}$ J/s, Solar Mass $M_\odot = 1.98 \times 10^{30}$ kg
- $1\text{AU} = 1.495 \times 10^{11}$ m
- Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}
- The mean molecular mass of H_2O is $18m_u$
- The atomic mass unit $m_u = 1.6605 \times 10^{-27}$ kg
- The Boltzmann constant $k_B = 1.3807 \times 10^{-23}$ J K^{-1}
- $M_\oplus = 5.98 \times 10^{24}$ kg, $R_\oplus = 6.378 \times 10^6$ m
- Specific heat of rock $c = 800$ J/kg/K, Melting temperature of rock $T_{melt} = 1000$K, $\rho_{rock} \approx 3000$ kg/m^3.
- Specific heat of ice $c = 1000$ J/kg/K, Melting temperature of ice $T_{melt} = 273$K
- Mass of Jupiter $M_J = 1.8986 \times 10^{27}$, Density of Jupiter $\rho_J = 1326$kg/m^3

1. Enceladus
   In 2005 the Cassini spacecraft made its first close passages with the Saturnian moon Enceladus, coming as close as 1,100-km from the satellite. The magnetometer onboard the Cassini spacecraft detected signatures of water vapour, suggesting that perhaps Enceladus retained an atmosphere of this material. Enceladus radius of $R = 252$km, mean density of $\rho = 1602$ kg m^3, and average surface temperature of 75K.

   a) Compute Enceladus' escape velocity.
   b) Compute the rms velocity of water vapour near the surface of Enceladus.
   c) Comment on Enceladus' ability to retain a water vapour atmosphere and/or the potential sources of the water.

   You can check your answers with the web page used in class (see the link).
   http://astro.unl.edu/naap/atmosphere/animations/gasRetentionPlot.html
   There is a ton of interesting information about this moon on wikipedia.

2. Atmospheric retention and Core Accretion of Gas Giant Planets

   a) Derive an expression for the escape velocity of a planet. Write three versions of the escape velocity, in terms of M & R, ρ & R, and in terms of ρ and M.

b) Using the atmospheric retention criteria, $\langle v^2 \rangle^{1/2} < v_{esc}/10$, write an expression for the minimum planet mass M required to retain a gas with molecular mass m. The expression should be a function of T,m, ρ, and physical constants.

c) In one scenario for the formation of gas giant planets, a rocky/icy body accumulates other condensed material via particle collisions until it grows large enough to gravitationally retain and attract gas directly from the protoplanetary gas disk that orbits the star.

Use values appropriate to a rocky protoplanet between the orbits of Jupiter and Saturn and compute the planet mass needed to retain molecular hydrogen H_2 (T = 100K, $\rho = 3000$kg m^{-3}). This can also be interpreted as the protoplanet mass needed to 'seed' the formation of a gas giant planet via a process known as 'core' accretion.

d) The binding energy of the protoplanet must be released in order to assemble it. Assume all the binding energy goes into heating the protoplanet and compute the temperature that the protoplanet is heated to, by its accumulation (see cw#8 for a discussion of this). You may ignore the melting phase change at $T = 1000K$.

e) How does this formation temperature affect the protoplanet's ability to retain molecular hydrogen? (use your expression from (a) to interpret your result).

f) How must the protoplanet evolve to retain a molecular hydrogen atmosphere?

g) In the core accretion scenario, how does the the mass of the 'core' needed for gas accretion depend on the location from the star? You may assume that the gas is in thermal equilibrium with small solid particles within the gas disk. Make a quantitative argument here by either using scaling arguments or working some numbers for orbital radii typical of extrasolar Hot Jupiters and discussing their implications.

h) What are the implications of the above result for the *in situ* formation of extrasolar Hot-Jupiters?

3. Jupiter's Formation and Contraction

a) Estimate the rate of energy output from Jupiter due to gravitational collapse alone, assuming the rate of gravitational energy release has been constant over the age of the solar system, that is

$$L_J \approx \frac{E}{\Delta t}$$

where E is the gravitational energy released (see Coursework #8) and $\Delta t = 4.5 \times 10^9$yr is the age of the solar system.

b) Compare your answer above with that net power output of Jupiter in you computed in Coursework #8 (i.e. the observed excess luminosity of 3.3×10^{17} Watts). What does this say imply about Jupiter's rate of energy output in the past?

4. Jupiter's Interior Structure

Jupiter is thought to have a rocky core deep in its interior, surrounded by an envelope of gas. We'll model this as a spherical planet with a core of one density ρ_c and a envelope of another, constant density ρ_e.

a) Derive an equation for the mass of the planet core of the two-component planetary model. Express your answer in terms of the core density and fractional radius fR where $f = R_c/R$ and R_c is the radius of the core.

b) Assume that Jupiter has a $10M_\oplus$ core and that the average density of the core is $\rho_c = 15000$kg/m^3. Determine f, the ratio of the core radius to the planet radius.

c) What is the average envelope density of the two component model?

d) Determine the moment-of-inertia ratio I/MR^2 for this two component model.

e) Compare your answer in part (d) to the measured value of the moment of inertia ratio for Jupiter ($I_J/M_J R_J^2 = 0.258$). What can you say about the mass distribution of Jupiter compared to the two-component model?

5. Exoplanet Detection

a) An extrasolar Earth-mass planet orbits a Sun-like star with an orbital radius of 1AU. The plane of the solar system lies in the line of sight. What is the velocity amplitude of the star's radial velocity variation caused by the planet and sketch the resulting stellar velocity vs. time curve.

b) If the planet were seen to transit its host star, what would be the fractional decrease in the stellar luminosity?

c) If the inclination of the orbit to the line of sight is not known (e.g. not in the most effective orientation for radial velocity detection), what is the probability that this planet would be seen transiting the star?