PHY 241 Planetary Systems - Coursework #8

Due: Tuesday, December 7, 2010 4pm

References:

- Ch. 21 An Introduction to Modern Astrophysics 2nd Edition Astronomy, by Carroll and Ostlie (several copies in the Library)
- Ch. 13 Planetary Sciences, DePater & Lissauer (copy in the library -scan on the course website). for exoplanets

Useful quantities:

- Solar luminosity $L_\odot = 3.8 \times 10^{26}$ J/s, Solar Mass $M_\odot = 1.98 \times 10^{30}$ kg
- 1AU $= 1.495 \times 10^{11}$ m
- Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}
- The atomic mass unit $m_u = 1.6605 \times 10^{-27}$ kg
- The Boltzmann constant $k_B = 1.3807 \times 10^{-23}$ J K^{-1}
- $M_\oplus = 5.98 \times 10^{24}$ kg, $R_\oplus = 6.378 \times 10^6$ m
- Specific heat of rock $c = 800$ J/kg/K, Melting temperature of rock $T_{melt} = 1000$K, $\rho_{rock} \approx 3000$ kg/m^3.
- Specific heat of ice $c = 1000$ J/kg/K, Melting temperature of ice $T_{melt} = 273$K
- Mass of Jupiter $M_J = 1.8986 \times 10^{27}$, Density of Jupiter $\rho_J = 1326$kg/m^3
- Specific heat of rock $c = 800$ J/kg/K, Melting temperature of rock $T_{melt} = 1000$K, $\rho_{rock} \approx 3000$ kg/m^3.
- Specific heat of ice $c = 1000$ J/kg/K, Melting temperature of ice at STP $T_{melt} = 273$K

1. Strength Part II: Internal pressure and shape
   The 'strength' of a material can be taken to mean the pressure a material can be subjected to before it begins to deform, flow or fracture. For geologic materials, material strength is generally measured in the lab. For example, lab results suggest that typical stony meteorites have a strength in the range of $S = 6 \times 10^7$ to 4×10^8 N m^{-2}. For water ice the compressive strength is roughly $S = 6 \times 10^6$ N m^{-2}.

   Here we'll calculate the internal pressure profile of spherical planet of uniform density ρ and examine for what size planet and at what depth the pressure is sufficiently large to overcome strength and the planet will flow to a spherical shape.

   a) Write down an expression for the gravitational acceleration $g(r)$ inside a planet. where r is the radial distance from the center of the planet.
   b) Using this answer and the hydrostatic assumption, derive a general expression for the pressure $P(r)$ inside the planet.
   c) Using the boundary condition that $P = 0$ at the surface ($r = R$), write down an expression for the pressure at the center of the planet in terms of ρ, G, and R.

d) The 'spherical' definition of a planet.
If the strength of rock is greater than the central pressure in a body, then nowhere inside does rock flow geologically. In this case the rock strength can maintain a highly irregular body shape. If nearly all of the body's interior is under pressure much greater than rock strength, then nearly the whole interior will flow geologically and the body will take on a largely spherical shape. Thus, the irregular surface shapes (e.g. mountains, gullies, craters,...etc.) are supported by rock strength and their size is limited by the size of the region where the internal pressure is less than the rock strength. If we set $S = P(r = 0.20R)$ (a somewhat arbitrary threshold) we can identify the maximum size of a body that will support a grossly irregular shape. Using this threshold, a rock strength of $S = 2 \times 10^8$ N m^{-2}, and the expression for $P(r)$, determine the maximum radius R_{max} of a rocky body that will support an irregular shape.

e) Alternatively, we can also examine the thickness of the body's outer shell where material strength can support irregular shapes (e.g. mountains). Using your expression for $P(r)$ write a relation for the radial distance r_S, below which the internal pressure is greater than the material strength $((P(r < r_S) > S))$.

f) The region where $r > r_S$ is dominated by rock strength and can support irregular shapes. How does the thickness of the strength dominated region (thickness $= R - r_S$) on Earth compare with the height of the largest mountain on Earth (Mauna Kea 10200m)?

g) What is the thickness of the strength dominated region on Mars? How does this compare with the height of Mars tallest mountain, Olympus Mons (23000m)?

2. Haumea's internal strength and structure
Haumea is one of the recently discovered dwarf planets in the Kuiper Belt. Its spectra suggests a surface of fairly clean water ice. It rotates rapidly and has an unusual light curve that may result from a non-spherical shape or spotted surface. Haumea has satellites which permit the determination of its mass ($M = 4.00 \times 10^{21}$ kg). Measurements of the reflected solar flux and re-radiated thermal flux in turn allow for the determination of Haumea's albedo, physical size and density. For the sake of this exercise consider Haumea as a sphere of uniform density $\rho = 2600$ kg/m^3

a) Compute the radius of Haumea and compare it with that of Pluto.

b) Considering that the strength of material making up Haumea is rock, how large should we expect the surface irregularities to be (i.e. how deep is the region where $P(r) < S$)? How do the size of these potential irregularities compare with the radius of Haumea? Note for this exercise you may simply reuse the expression derived in previous problems.

c) If Haumea's strength is that of rock, how large might the surface irregularities be (i.e. how thick is the 'strength dominated' region near Haumea's surface)?

d) If ice has a density of $\rho_{ice} = 1000$ kg/m^3 and rock as a density of $rho_{rock} = 3000$ kg/m^3, what is the fraction of Haumea's mass that is due to rock?

3. Planetary Binding Energy

a) Show that the total energy released when building a body with mass M, radius R, and bulk density (ρ) by impacting planetesimals is at least:

$$E_b = -\frac{3GM^2}{5R} = -\frac{3GR^5}{5}\left(\frac{4\pi}{3}\rho\right)^2 \tag{1}$$

It may be convenient to use the definition of density for a sphere and work with ρ and r.

b) Assuming all this energy goes into heat, write an expression for the temperature rise of the planet. Express your answer in terms of density and radius. Solve this to obtain an expression for the minimum size a body must grow to in order to reach the melting temperature T_{melt} by this process?

c) For bodies made of rock compare this with the radius with the size of the inner planets and asteroids (e.g. how does this size compare with Earth, Mars, Ceres . . . etc.) and comment on their plausible melting due to release of gravitational energy during formation.

d) Repeat the above calculation for icy bodies and compare this radius to the sizes of the solar system's icy satellites (e.g. Ganymede, Callisto, Enceladus, Mimas, Ariel,...etc) and to the largest Kuiper Belt Objects (e.g. Pluto, Haumea,...etc). (The Astronomy Workshop 'Planet/Satellite Caluculators' may be useful for comparing your answer to the size of bodies). Which may have undergone global melting via release of gravitational energy?

e) Data returned by the Galileo mission to Jupiter (1995-2003) suggests that Callisto is not entirely differentiated, and therefore has never been entirely molten. Given your calculations above, discuss how Callisto might have avoided global melting during its assembly.