PHY 241 Planetary Systems - Coursework #7

Due: Tuesday, November 30, 2010 4pm

References:

- Ch. 7 Hartmann, W.K. 'Moons & Planets' Interplanetary Worldlets: Asteroids and Comets.

Useful quantities:

- Solar luminosity $L_\odot = 3.8 \times 10^{26}$ J/s, Solar Mass $M_\odot = 1.98 \times 10^{30}$ kg
- 1AU $= 1.495 \times 10^{11}$ m
- Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}
- Mass of Jupiter $M_J = 1.8986 \times 10^{27}$, Density of Jupiter $\rho_J = 1326$kg/m^3, Jupiter's semimajor axis $a_J = 5.20$AU.

1. Albedo and Flux [4 marks]

   a) Write an expression for the rate that solar energy is incident on a body a distance r_p from the Sun. This expression should be a function of the solar luminosity ($L_\odot$), the body's radius (R) and its distance from the Sun (r_p).

   b) Write an expression for the energy flux reflected by a planet observed at Earth ($F_{reflect}$) a distance d from the planet (Note: $d \neq r_p$ - draw yourself a diagram of the situation of this is not obvious). Note that the body only reflects from the side facing the Sun, not from the whole surface of the body.

   c) Write an expression for the energy flux absorbed by the body, re-emitted (assuming it is a blackbody, i.e. $A_{RE} = 0$) at longer wavelengths and observed at Earth ($F_{thermal}$).

   d) Derive an expression for the ratio $F_{reflect}/F_{thermal}$ and show is only a function of the body's albedo (A_v) and that

$$\frac{F_{reflect}}{F_{thermal}} = \frac{2A_v}{(1 - A_v)} \tag{1}$$

2. Estimating Eris' size [4 marks]
   The newly discovered dwarf planet Eris has an orbital semi-major axis of $a = 67.7AU$ and eccentricity $e = 0.441$. It is currently near its aphelion.

   a) What are the aphelion and perihelion distances for Eris?

   b) Observations of Eris at aphelion obtain $F_{reflect} = 8.37 \times 10^{-16}$ J m^{-2} s^{-1} and $F_{thermal} = 5.58 \times 10^{-16}$ J m^{-2} s^{-1}. What is Eris' albedo?

   c) Using the observed flux and Eris albedo, estimate Eris' radius. Assume that we are observing Eris at opposition (i.e. the Sun, Earth and Eris lie in a line with Earth between the Sun and Eris.).

   d) What are some of the approximations of this model that might introduce error in this estimate of Eris' size? [Also see links on the module web page to stories on estimating Eris' size. Some are as recent as a few weeks ago].

3. Strength - Part I: Rotation [3 marks]

a) Calculate the minimum rotation period for a strengthless spherical planet. Express your final answer in terms of density ρ and the gravitational constant G (although using the mass and radius M and R may be useful in intermediate steps).

b) Use the value calculated above, the density of common rock ($\rho_{rock} \approx 3000$ kg m^3) and the figure of rotation rates vs. diameter for asteroids to interpret their physical properities. For which range of sizes is gravity most important for holding asteroids together? For which range of sizes is physical strength most important for holding asteroids together?

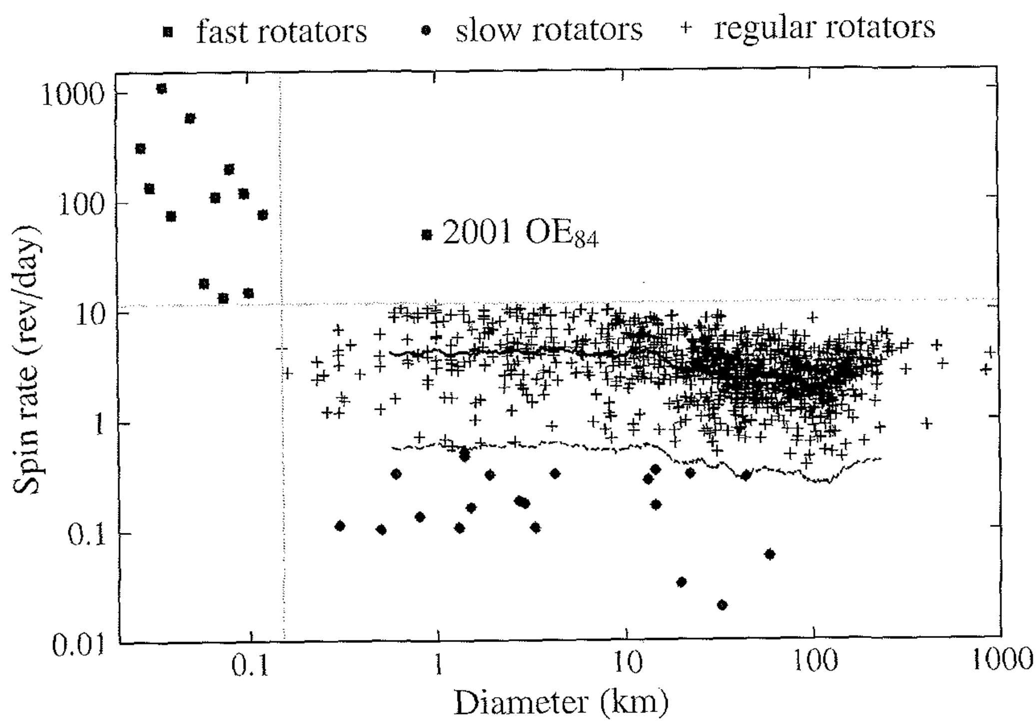

Figure 14.7. Asteroids in the (diameter, spin rate) plane, in logarithmic units. The thick line shows the average spin rate for objects larger than 1 km. The shallow minimum at ≃ 100 km corresponds to the "angular momentum drain" process: when the velocities of the fragments from an impact are near the escape velocity, more objects escape in the prograde sense relative to the rotation of the asteroid, and with their recoil slow down the spin. Full circles below the dashed line, at three standard deviations from the mean, are slow rotators, with unusually long periods, up to a few months. Full squares on the upper left are fast rotators – small near-Earth asteroids with periods as short as a few minutes. The horizontal line at ≃ 12 rev/day, a

FIGURE 1: Asteroid rotation rates (revolutions/day) vs. Diameter (km). The data was obtained by examining the frequency of the light curve variations for each body. Note the logarithmic scales.

4. Taking Jupiter's temperature [5 marks]

a) Jupiter has an estimated albedo of 0.36. Compute the mean power in Watts emitted by Jupiter assuming the giant planet is only reemitting light absorbed from the Sun.

b) Jupiter is observed to emit like a blackbody with an effective temperature 124 K. What measurements must be made to determine this 'effective temperature' of Jupiter's thermal emission.

c) Compute Jupiter's emitted power on that basis (recall, for a blackbody, the emitted flux is σT^4 and assume Jupiter is rotating rapidly, emitting evenly from its entire surface).

Notice that the emitted power is larger than what you found in part (a)–Jupiter radiates more energy than it receives from the Sun!

d) Compute the net power output of Jupiter. This is the difference between the power emitted (b) and the power absorbed (a).

e) Where might this heat surplus originate from?