PHY 241 Planetary Systems - Coursework #6

Due: Tuesday, November 23, 2010 4pm

References:

- Sections 6.6 & 7.6 of Physical Processes in the Solar System (Landstreet) for discussions of luminosity, flux, albedo and derivations of different approximate temperatures.
- Ch. 2 Introductory Astronomy & Astrophysics (hereafter IAA), by Zeilik, Gregory and Smith (several copies in the Library). This text has a detailed discussion of the albedo and equilibrium temperatures. Check the course website for additional information.

Potentially useful quantities:

- Solar luminosity $L_\odot = 3.8 \times 10^{26}$ J/s
- 1AU $= 1.495 \times 10^{11}$ m
- Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}
- The mean molecular mass of H_2O is $18m_u$
- The atomic mass unit $m_u = 1.6605 \times 10^{-27}$ kg
- The Boltzmann constant $k_B = 1.3807 \times 10^{-23}$ J K^{-1}

1. Deriving the equilibrium temperature [8 marks]
   Consider a spherical body with radius R, visual albedo A_v orbit a star of luminosity L_* at a distance r.

   a) Using the Stefan Boltzmann Equation and first principles (e.g. concepts of flux, luminosity,...etc.), show that the total heat input per second, the power absorbed (P_A), by the body is
   $$P_A = (1 - A_v)\left(\frac{L_*}{4\pi r^2}\right)\pi R^2. \quad (1)$$

   b) On absorbing this energy the surface heats up. The surface of the body re-radiates as a blackbody. If the body rotates quickly (i.e. faster than it reradiates), energy is radiated away from the object's entire surface. If the body rotates slow enough that energy is reradiated from the illuminated side before completing a rotation; energy is radiated from little more than half the object's surface. Assuming the body rotates rapidly show that the total power radiated by the body each second is
   $$P_{RE} = (1 - A_{RE})\sigma T^4(4\pi R^2). \quad (2)$$
   Here A_{RE} is the albedo in the re-emitted wavelength. Recall that a blackbody is both a perfect absorber and perfect emitter. The albedo quantifies a body's inefficiency both in absorbing *and* re-emitting.

   c) How does the wavelength of maximum intensity of the absorbed and reradiated energy compare with the wavelength of maximum intensity of the incident energy? Which wavelength is longer or shorter (i.e. bluer or redder)? Why?

   d) Assuming the body is in thermal equilibrium, derive an expression for the equilibrium temperature T_{eq} in terms of L_*, r, A_v, and A_{RE}.

e) Using appropriate numerical constants show that when the temperature is measured in Kelvin and the distance in AU the temperature of a body orbiting the Sun is about

$$T_{eq} = 280 \frac{(1 - A_v)^{1/4}}{(1 - A_{RE})^{1/4}} \left(\frac{1 \text{ AU}}{r} \right)^{1/2} \text{ K} \tag{3}$$

f) Assuming the visual albedo (A_v) is similar to the reradiating albedo (A_{RE}), calculate the equilibrium surface temperature of Venus.

g) Venus' actual surface temperature is about 750K. Using the terms in Equation (3) comment on how any discrepancy with the equilibrium temperature might arise.

2. The Habitable Zone [4 marks]
The presence of liquid water is thought to be one of the essential criteria for a planet (or moon) to be suitable for life to develop (i.e. is 'habitable'). This question explores some of the issues that determine where liquid water might exist on a planet's surface. Water's boiling and freezing points provide the rough thermal boundaries of the so-called Habitable Zone (HZ) around a star.

a) Using the expression for the equilibrium temperature at a planet's surface, calculate the orbital radii about the Sun where
   i. water freezes at standard temperature and pressure (STP).
   ii. water boils at standard temperature and pressure (STP).

b) Compare the boundaries of the HZ to the locations of the Solar System's planets. Which planets are 'habitable', by this criteria?

c) Using the symbolic expression for the equilibrium temperature above, discuss qualitatively how the boundaries of the HZ change when considering:
   i. Planets orbiting stars other than the Sun (i.e. stars of different sizes and surface temperatures)?
   ii. Planets with varying reflection and or emission efficiencies (i.e. albedoes)?

d) Could a planet residing *outside* the HZ sustain a reservoir of liquid water? Defend your answer.

3. The Snow Line [2 marks]
The planets and small bodies of the Solar System are thought to have originated in a gas disk orbiting the Sun. The rocky and icy material condensed out of this gaseous disk, going directly from gaseous to solid thermodynamic state. At very low pressures, such as those in the solar system's protoplanetary nebula H_20 condenses at a temperatures below about 177K (See attached phase diagram for water. Although it doesn't go down to such low pressures as appropriate for the solar nebula - you can see the validity of this choice for the condesation/sublimation temperature by extrapolating).

a) Assuming particles of similar reflecting and absorbing properties (i.e. albedoes), above what orbital radius is the nebula cool enough for water ice grains to condense? List your answer in AU. This is the so-called 'snow-line'.

b) Discuss whether water ice can condense from the gaseous phase to the solid phase (as in a protoplanetary nebula) at 1AU. What does this imply as the source of the water in Earth's oceans? Did this material originate near Earth's present orbit?

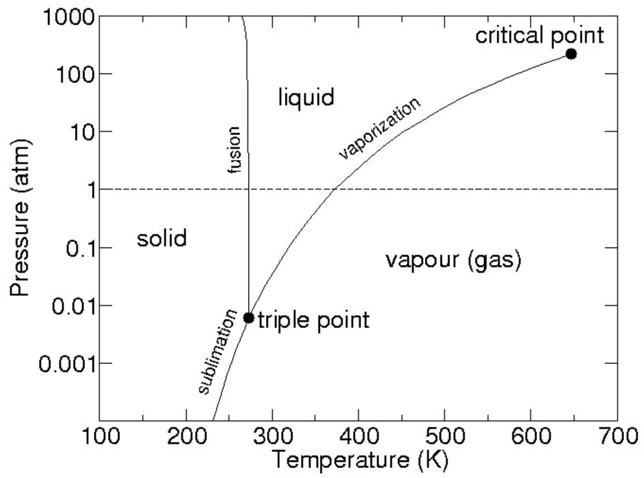

FIGURE 1: Phase diagram for H_2O. Pressure vs. Temperature. Note this doesn't go down to the low pressures appropriate to the solar nebula.