PHY 241 Planetary Systems - Coursework #5

Due: Thursday, November 4, 2010 4pm

Some physical data that might be useful:

Object	Mass (kg)	Radius (km)	Rotation Period	Semi-major axis
Sun	1.98×10^{30}	6.96×10^{5}	~ 25 days	
Earth	5.97×10^{24}	6,378	23.93 h	1.00 AU
Moon	7.35×10^{22}	1,737	synchronous	384,400 km
Jupiter	1.90×10^{27}	71,398	9.92 h	5.203 AU
Saturn	5.68×10^{26}	60,330	10.66 h	9.537 AU
Mimas	3.85×10^{19}	199	synchronous	185,520 km

1. The magnitude of the orbital angular momentum of a satellite of mass m_s orbiting a planet of mass M_p with semi-major axis a and eccentricity e is (coursework #2):

$$L = m(GM_p a)^{1/2}(1-e^2)^{1/2} \qquad (\text{for} \quad m_s \ll M_p) \tag{1}$$

Using the definition of a torque ($\mathbf{\Gamma}$),

$$\mathbf{\Gamma} = \frac{d\mathbf{L}}{dt} \tag{2}$$

and assuming the masses are constant and the orbit circular, derive a relation between the magnitude of the torque applied to a satellite and the rate of change of its semi-major axis (da/dt). [1 mark]

2. The torque on a satellite from the tidal bulge which it raises on its primary is given by

$$\Gamma = \frac{dL}{dt} = \frac{3}{2}\left(\frac{k_{2p}}{Q_p}\right)\frac{Gm_s^2 R_p^5}{r^6}\text{sign}(\Omega_p - \omega_s) \tag{3}$$

where R_p and Ω_p are the radius and rotational angular velocity of the primary and m_s is the mass of the satellite, ω_s is its orbital angular velocity and r is the distance between the primary and the satellite. [3 marks]

   a) How does the following term

$$\text{sign}(\Omega_p - \omega_s)$$

   in Eq. (3) affect the sign and magnitude of the tidal torque?

   b) Equating the tidal torque on the satellite (Eq. 3) with the expression you derived relating dL/dt and da/dt above, and considering a circular orbit ($r = a$), find an expression for the semi-major axis of the satellite as a function of time and its initial semi-major axis a_o at some initial time t_0.

3. Mimas is the regular satellite of Saturn that orbits closest to, but above Saturn's corotation orbit [see e.g. coursework #1] [5 marks]

   a) Draw a diagram of a top down view of the Saturn-Mimas system which shows the following:

      i. Saturn's rotation.
      ii. Mimas' circular orbit about Saturn.
      iii. Saturn's corotation orbit (i.e. the orbit synchronous with Saturn's rotation).
      iv. The Roche radius for a body with Mimas' density.

v. The tidal bulges that Mimas raises in Saturn.

b) In a few sentences and using the diagram you've constructed above, explain how the net force from the bulges that Mimas raises in Saturn exerts a torque on Mimas, causing its orbit to expand.

c) What is the closest distance to Saturn that Mimas could have formed, such that it has always tidally evolved outward? [you may assume Saturn's current rotation rate is constant.]

d) Saturn and Mimas probably formed about 4.5×10^9 years ago. Use the expression for the semi-major axis as a function of time and Mimas' smallest (computed above) and current orbits to estimate a value of the ratio (k_{2p}/Q_p) for Saturn.

4. The age of the solar system is about 4.5×10^9 years. Saturn and its system of regular satellites were probably completely formed within the first $10^6 - 10^7$ years of solar system history. The mass of Saturn's ring system has a lower limit of about the mass of Mimas. Here we use a few calculations to evaluate how a primordial satellite close to Saturn might evolve through the Roche zone. [3 marks]

a) Using the value of (k_{2p}/Q_p) for Saturn you've estimated above, compute the time it would take a satellite with a mass equal to that of Mimas, to evolve inward from the synchronous orbit to Saturn's surface? List your answer in seconds and years.

i. How much of the solar system's history could such a satellite persist?

ii. If this satellite were 4x more massive, by what factor would the time required to reach Saturn's surface change?

b) How would tidal forces affect this satellite during its orbital decay to Saturn's surface? What are the possible fates of the satellite?

c) What do your answers to the above questions imply for constraining when Saturn's ring system may have formed via this process?