PHY241 Planetary Systems - Coursework #3

Due: Tuesday, October 19, 2010 4pm

Some physical data that might be useful:

Object	Mass (kg)	Radius (km)	Rotation Period	Semi-major axis
Sun	1.98×10^{30}	6.96×10^{5}	~ 25 days	
Mercury	3.30×10^{23}	2440	1047.51 h	0.3871 AU
Earth	5.97×10^{24}	6,378	23.93 h	1.00 AU
Moon	7.35×10^{22}	1,737	synchronous	384,400 km
Mars	6.41×10^{23}	3,394	24.64 h	1.523 AU
Phobos	1.08×10^{16}	11.2	synchronous	9380 km
Jupiter	1.90×10^{27}	71,398	9.92 h	5.203 AU
Saturn	5.68×10^{26}	60,330	10.66 h	9.537 AU
Mimas	3.85×10^{19}	199	synchronous	185,520 km

Additional values needed can be found on the web (wikipedia is a good source).

1. Planning spacecraft missions - Part I [10 marks]
   The Mercury Messenger spacecraft in en route to Mercury and will begin orbiting the planet later this year. This problem examines some issues related to getting to Mercury. For simplicity consider that both Earth and Mercury on circular orbits. After launch the spacecraft is on a circular orbit with a semi-major axis equal to the Earth's and is no longer 'near' the Earth in its orbit (i.e. you may now ignore gravitational interaction with Earth). It will need to 'do a burn' to change it's speed to reach Mercury's orbit. Treat the burn as an instantaneous change in speed or Δv.

   a) The new, 'post burn' transfer orbit must take the spacecraft from Earth's orbit to Mercury's (i.e. it must cross both). Make a diagram showing Earth's and Mercury's orbit and identify the pericentre and apocentre of the transfer orbit (consider the orbit that requires a minimum of energy change - as discussed in class).
   b) Compute the semi-major axis and eccentricity of the transfer orbit.
   c) Compute the speed of the spacecraft just after the burn.
   d) Compute the Earth's circular orbital velocity and the Δv needed to go from the Earth orbit to the transfer orbit.
   e) What is the spacecraft's velocity as it reaches Mercury's orbit?
   f) At what relative velocity does the spacecraft approach Mercury's orbit? (i.e. at what velocity does the spacecraft approach Mercury - ignore Mercury's gravity for the time being).

2. Collinear Lagrange Points [9 marks]
   In class we derived the distance to the L1 Lagrangian Point and found it to be a distance

$$r_{Hill} = a_s \left(\frac{M_s}{3M_p} \right)^{1/3} \tag{1}$$

   interior to the planet along the Sun-planet line, where the 's' subscript refers to the secondary and the 'p' subscript refers to the primary.

   a) Derive an approximate expression for the distance from the planet to the exterior L2 Lagrangian equilibrium point.

b) To the level of this approximation how does the distance from the planet to the L2 compare with the distance from the planet to the L1 point?

3. Tidal length scales - the size of things [10 marks]

   a) Describe briefly what the Hill sphere tells you.
   b) Using the mass of the Milky Way galaxy ($5.8 \times 10^{11} M_\odot$, and the distance of the Sun from the galactic center (26,000 light years), compute the size of the Sun's Hill sphere with respect to the galactic center. Express your answer in AU.
   c) Alpha Centauri is the nearest star system to the Sun. See the wikipedia site for a description. http://en.wikipedia.org/wiki/Alpha_Centauri.
      If the total mass of the stars is about 2-$M_\odot$ and they are 4.37 light years away from the Sun, what is the Hill sphere of the Sun with respect to Alpha Centauri?
   d) Which mass is more important for determining the effective 'Hill sphere' of the Solar System (the galaxy or Alpha Centauri)?
   e) How do these estimates for the solar system's Hill sphere compare with the size of the solar system's exterior 'Oort' comet cloud? (you can find the mass of the galaxy and gross characteristics of the Oort cloud on Wikipedia).

4. Hill sphere and Roche Radius [8 marks]

   a) Compute the Hill sphere of Mars' satellite Phobos with respect to Mars and express this Hill sphere in Phobos radii.
   b) Compute the Roche radius for bodies orbiting Mars with density $\rho = 1900$ kg/m^3. Express the answer in metres and Martian radii.
   c) Compute the Hill sphere of an icy body ($\rho = 1000$ kg/m^3) of 10 metres in radius, orbiting Saturn at 1.2 Saturnian radii.
   d) Imagine Jupiter spiralling in towards the Sun. At what distance from the Sun would Jupiter's surface, just touch it's Hill sphere? What is the orbital period here?

5. Stability and Dynamics Near Lagrange Points
   In class we've discussed the locations of the collinear Lagrange points and briefly mentioned the Trojan or equilateral Lagrange Equilibrium points in passing.[1]

   Here you will use the 'Lagrange Point Explorer' numerical integrator from the 'Astronomy Workshop'[2] to explore the stability and character of orbits near the Lagrange points of Jupiter.

   Note: the integrator uses a rotating frame where the x-axis is along the line joining the planet to the star (with the + direction to the right, towards 3-O'clock), the y-axis is perpendicular to this and in the orbital plane (with the + direction up, towards 12-O'clock), and the $+z$-axis is out of the page.

   a) Compute the Hill radius of Jupiter and express your answer both as a fraction of Jupiter's semi major axis (i.e. r_H/a_J) and in AU. This provides a length scale for the problem.

      We will use following parameters for the Sun-Jupiter system to examine qualitatively the dynamics near the equilibrium points.

[1] See *Solar System Dynamics*, chapter 3 by QMUL's Prof. Carl Murray for a complete derivation of the location and stability of the Lagrange points. You might also take the MSc/MSci module ASTM001 Solar System for a more complete treatment in a later year of your programme.

[2] These integrators can be found at http://janus.astro.umd.edu/AW/awtools.html#integrators

Mass of Central Body	1.000	Planet Mass (in central units)	0.000954
Planet semi major axis	5.210	Planet eccentricity	0.000
x (AU)	0.000	v_x (km/s)	0.000
y (AU)	0.000	v_y (km/s)	0.000
z (AU)	0.000	v_z (km/s)	0.000

These will place the test mass at the location of the equilibrium point (chosen from the drop down menu). We will explore the behavior near these equilibrium points by considering initial conditions that start with small deviations from the equilibrium points in either **r** or **v**.

Also choose model parameters of '1e-14 Most Accurate', integration time of '50' orbital periods with an output interval of '6' days. These values are all pretty close to the default values.

When plotting, use an 'x-y' plot in the 'rotating' frame.

b) Write a few sentences to answer the questions below. Save and turn in a few of the orbits you produced to justify your answers.

   i. **Characterizing the L1** - Try simulations with particles starting at the L1 point. Vary the initial position along the x-axis and examine how this affects the orbit? How does varying x in the range [-.02 and +0.02] AU affect the region traversed by the particle?
   Does the particle stay 'near' the L1 throughout the calculation and oscillate about this location (i.e. is this a stable equilibrium point)?

   ii. **Characterizing the L4** - Perform simulations with particles starting at the L4 or L5 points. Vary the initial velocity and examine how this affects the orbit.
   Does the L4 appear to be a stable equilibrium point? For example, try an initial condition very close to the equilibrium point, (e.g. $v_x = 0.001$km/s with all other values equal to zero). Does the particle stay 'near' and oscillate about the L4 throughout the calculation for a very small deviations from it?
   How does increasing v_x in the range [0.1,0.2,0.3,0.4] km/s affect the character of the orbit? What happens when v_x is much larger (e.g. 2km/s)?

A few other topics you could explore . . . [but do not have to turn in].

- What are the stability properties and orbital character near the L2, L5 points. How do these compare with the L1 and L4?
- How does the planet's eccentricity (which we've taken as zero up to this point) affect the orbits near the Lagrange points?
- How does the planet's mass affect the velocity needed to escape the L4 or L5 equilibrium points? Are Mars' L4 and L5 points more dynamically fragile than Jupiter's?

Food for thought: A good MSci/BSc project would be to address the a few of the following (unanswered) questions. Why do some planets and satellites have Trojan companions at the L4 and L5 (e.g. Jupiter, Neptune, Mars, Tethys, Dione, Janus and Epimetheus) while others do not? Should extrasolar planets have Lagrangian companions? How does the presence or absence of these Lagrangian companions and their properties inform our understanding of origin and evolution of the solar system? A project of this sort would involve a bit of theory and some modelling like that in this coursework.