PHY241 Planetary Systems - Coursework #2

Due: Tuesday, October 12, 2010 4pm

References:

- Ch. 1 Physical Processes in the Solar System, Landstreet
- Ch. 2 Introductory Astronomy & Astrophysics (hereafter IAA), by Zeilik, Gregory and Smith (several copies in the Library)
- Ch. 2 & 3 Moons & Planets, Hartmann (hereafter, just Hartmann).
- Note: You may use a spreadsheet to calculate values for the planets. Alternatively the Planet & Satellite Calculator website, by Doug Hamilton, has many parameters pre-loaded and may be useful for doing the calculations.
  http://janus.astro.umd.edu/AW/awtools.html#calculators
  Please do not do very repetitive calculations by hand. Ask if you have questions.

Some physical data that might be useful:

Object	Mass (kg)	Radius (km)	Rotation Period	Semi-major axis
Sun	1.98×10^{30}	6.96×10^{5}	~ 25 days	
Jupiter	1.90×10^{27}	71,398	9.92 h	5.203 AU
Saturn	5.68×10^{26}	60,330	10.66 h	9.537 AU
Neptune	1.028×10^{25}	24,764	16.1 h	30.06 AU

1. The Vis-viva Equation [4 marks]

   a) Use the energy relation and the case of a circular orbit to derive the vis-viva Equation.

   $$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right) \tag{1}$$

   b) Is this relation valid for eccentric orbits as well? Why? or Why not?

2. Orbital angular momentum

   a) Use the vis-viva equation and the expression for the apocenter to write the orbital angular momentum L of an orbit in terms of the masses, semi-major axis and eccentricity. [3 marks]

   b) How does angular momentum scale with orbital semi-major axis (i.e. $L \propto a^n$ what is n)? [1 mark]

3. Angular momentum in the solar system.

   a) Treating the Sun as a sphere of uniform density, calculate its *rotational* angular momentum. [1 mark]

   b) Compute Jupiter's *orbital* angular momentum [1 mark]

   c) Which has greater orbital angular momentum

      i. Jupiter or Saturn? [1 mark]

      ii. Saturn or Neptune? [1 mark]

   d) Where is most of the solar system's angular momentum? [1 mark]

4. The synchronous or co-rotation orbit plays an important role in the tidal evolution of satellites and planets. We will discuss tides and their implications later in the module.

   a) Derive a general expression for the semi-major axis of a planetary satellite whose orbital period about the planet is equal to the planet's rotation period. This is the so-called co-rotation orbit (a_c). [4 marks]

   b) Compute the semi-major axis of the corotation orbit for each of the terrestrial and giant planets. Express your answer in both meters and planetary radii. [2 marks]

c) Compute corotation radius of the Sun, listing your answer in terms of solar radii and AU. How does this compare with Mercury's orbit? Is it comparable in size? Is it much much smaller? [1 mark]

5. The observational discovery of the planet Neptune was first announced on 23 September 1846. On what date did/will Neptune complete its first orbit since discovery? [1 mark]