1 MT1 Homework Problems: Week 1

1) Differentiate the following with respect to x
i) $\sin(x)\cos(\pi x)$,
ii) $(ax-b)^n$,
iii) $\sin^2(\pi x)$,
iv) $\ln[\cos(x)]$,
v) $\log_{10}(x^2+1)$,
vi) $e^{ax}\sin(x)$,
vii) $\frac{x^2}{(1+x)^2}$,
viii)$\sin^3\left(\frac{x}{2\pi}\right)$,
ix) $\frac{e^x\sin(x)}{x^2\cos(x)}$,
x) $\ln(\pi x)$.

2) Evaluate by <u>implicit</u> differentiation
i) $\frac{\mathrm{dy}}{\mathrm{dx}}$ when $x^3+y^2=1$,
ii)$\frac{\mathrm{dy}}{\mathrm{dx}}$ when $3x^2+xy^2-9=0$.

3) If $x=\gamma\sin(\theta)$, and $y=\cos(\theta)$, evaluate $\frac{\mathrm{dy}}{\mathrm{dx}}$, and $\frac{\mathrm{d^2y}}{\mathrm{dx^2}}$.

4) Find the angle between $y=x^2$, and $y=x$ at $x=1$.

5) Find the equations of the tangent to the curve defined by $y=\sin(\theta)$, and $x=\cos(\theta)$ when $\theta=\frac{\pi}{4}$.

6) Prove that $x=e^{-\beta t}\cos(\omega t)$ satisfies $\frac{\mathrm{d^2x}}{\mathrm{dt^2}}+A\frac{\mathrm{dx}}{\mathrm{dt}}+Bx=0$, thus determine A and B.

7) Prove that $y=A(\sin(\omega x)+\cos(\omega x))$ satisfies $\frac{\mathrm{d^2y}}{\mathrm{dx^2}}\propto \mathrm{y}$, and determine the constant of proportionality.

2 MT1 Homework Problems: Week 2

1) Evaluate the following integrals
i) $\int \alpha \cos(\pi x)\mathrm{dx}$
ii) $\int \frac{1}{\pi} e^{\pi x}\mathrm{dx}$
iii) $\int x \cos(\pi x)\mathrm{dx}$
iv) $\int \tan(\pi x)\mathrm{dx}$

2) Integrate $e^{\pi x}\cos(\pi x)$ with respect to x, by parts.

3) Integrate the following.
i) $x \ln|x|$
ii) $\sin^2(x - \pi/6)$
iii) $x \sin(\pi x)$

4) Evaluate $\int \frac{1}{(x-1)(2x+1)}\mathrm{dx}$

5) Evaluate
i) $\int\limits_0^{\pi} \cos^2 x\mathrm{dx}$
ii) $\int\limits_0^{1/2} x \sin \pi x\mathrm{dx}$

6) Evaluate $\int\limits_0^{+\infty} e^{-x}\mathrm{dx}$

3 MT1 Homework Problems: Week 3

1) Differentiate the following with respect to x
i) $\pi^2 y = \arctan(\alpha x)$,
ii) $\beta y = \arccos(\beta x)$, where α and β are constants.

2) Evaluate x in the interval $0 < x \leq 2\pi$ for the following
i) $x = \arctan \frac{1}{\sqrt{5}}$
ii) $x = \arccos \frac{1}{\sqrt{2}}$
iii) $x = \arcsin -\frac{\sqrt{3}}{2}$

3) Determine the stationary points (the turning points and points of inflection) of the function $y = x + \sin(x)$ in the interval $0 < x \leq 2\pi$. Sketch the function $y(x)$, indicating the x and y co-ordinates of turning points on the sketch.

4) A damped oscillator has a displacement x as a function of time t which is given by $x = e^{-t}\cos(\pi t)$. Find the time at which the displacement is a maximum, when it is a minimum, and also find the time when x is a point of inflection.

5) Show that $y = xe^x$ satisfies

$$A\frac{\mathrm{d^2y}}{\mathrm{dx^2}} + B\frac{\mathrm{dy}}{\mathrm{dx}} + Cy = 0,$$

hence determine a general solution for A, B, and C.

6) Find the radius of curvature and the position of the center of curvature of the function $y = x \ln x$ at $x = 1$.

You may need to recall that the radius of curvature of a function $y(x)$ is given by

$$R = \frac{\left[1 + \left(\frac{\mathrm{dy}}{\mathrm{dx}}\right)^2\right]^{3/2}}{\frac{\mathrm{d^2y}}{\mathrm{dx^2}}}.$$

4 MT1 Homework Problems: Week 4

1) Find all of the first and second partial derivatives of
i) $z = e^x + y^2$.
ii) $z = y\sin(x) + y^2$.
iii) $z = e^{-ET}$.
iv) $\sin(\alpha x + \beta t)$, where α and β are constants.

2) Show that $z = e^x(x\cos y + y\sin x)$ satisfies

$$\frac{\partial^2 \mathrm{z}}{\partial \mathrm{x}^2} + \frac{\partial^2 \mathrm{z}}{\partial \mathrm{y}^2} = 2e^x(\cos y + y\cos x).$$

3) Two masses m_1 and m_2 are placed near each other in a Cavendish experiment, whose aim is to measure the gravitational constant G. The position of the first mass is given by x_1, and the position of the second mass is given by x_2 on some scale. Calculate the fractional change in the force between the masses $\delta F/F$ due small changes in x_1 and x_2 given that

$$F = \frac{Gm_1m_2}{(x_1 - x_2)^2}$$

4) Given that $z = xy^2 + x$, where $x = R\cos\theta$ and $y = R\sin\theta$, calculate $\frac{\mathrm{dz}}{\mathrm{d}\theta}$ in terms of θ.

5) Find $\frac{\partial \mathrm{z}}{\partial \theta}$ and $\frac{\partial \mathrm{z}}{\partial \phi}$ in terms of x and t for $z = \sin(kx - \omega t)$, where $\theta = x + t^2$ and $\phi = x^2 - t$. k and ω are constants.

6) Given that $z = \ln|x|\sin(y)$ where $x = uv^2$ and $y = v\sin u$, calculate $\frac{\partial \mathrm{z}}{\partial \mathrm{u}}$ and $\frac{\partial \mathrm{z}}{\partial \mathrm{v}}$.

5 MT1 Homework Problems: Week 5

1)

i) Find the limiting value of

$$\frac{3x^3 + 4x + 1}{7x^3 + 2x^2 + 1}$$

as $x \to \infty$.

ii) Find the limiting value of

$$\frac{x^5 + 4x^3 + x}{3x^5 + 2x}$$

as $x \to \infty$.

2) Estimate $\sqrt{0.8}$ using the first four terms of a Binomial series expansion.

3) Determine a Taylor series expansion for $\cos(2x)$ about the point $x = \pi/8$, hence determine a series expression for $\cos^2(2x)$.

4) Determine Maclaurin series expansions for e^{Ax} and $\sin(\pi x)$. Hence determine a expression for $e^{2x}\sin(\pi x)$.

5) Write down the Maclaurin series expansion for $f(x) = e^{-\gamma x}$. Using the first four terms of the expansion estimate $f(x)$ when $x = \ln 2/\gamma$.

6 MT1 Homework Problems: Week 6

1) Simplify the following, expressing the complex numbers in the form $a+bi$:
i) $(5+22i)-(-3-1i)$,
ii) $(5+22i)\times(-3-1i)$,
iii) $\frac{5+22i}{-3-1i}$,
iv) $e^{i\pi/2}$.

2) Simplify the following, expressing the complex numbers in the form $re^{i\theta}$:
i) $(1+2i)-(-2+1i)$,
ii) $(1+2i)\times(-2+1i)$,
iii) $\frac{1+2i}{-2+1i}$,
iv) $5\cos(2\pi)+5i\sin(2\pi)$.

3)
i) If

$$z=\frac{1}{11i}+\frac{1}{2-1i},$$

write z in the form $a+bi$
ii) For $z=1+1i$, determine z^3, $\sqrt{z}$, and zz^*
iii) Find expressions for $\cos\theta$ and $\sin\theta$ in terms of $e^{i\theta}$.

4) Write the following complex number in the form $a+bi$:

$$z=\frac{(M_{11}-M_{22})-(i/2)(\Gamma_{11}-\Gamma_{22})}{\Delta m-(i/2)\Delta\Gamma},$$

where M_{ij}, Γ_{ij}, Δm, and $\Delta\Gamma$ are constants, and $i,j=1,2$.

5) Find the square roots of $z=3$ and draw these on an Argand diagram.

6) Find the fourth roots of $z=1$ and draw these on an Argand diagram.

7) Determine the Maclaurin series expansion for $\cosh(x)$.

7 MT1 Homework Problems: Week 7

1) Determine the reduction formula for $I_n = \int x^n e^{-x}\mathrm{dx}$, hence evaluate $I_3 = \int x^3 e^{-x}\mathrm{dx}$.

2) y is given by $1+\cos(\pi x)$. Calculate $\langle y \rangle$ over the interval $x = 0$ to $x = 1/2$.

3) By suitable integration, calculate the area of an ellipse given by $x = 2\cos\theta$, $y = \sin\theta$ between $\theta = 0$ and $\theta = \pi/4$.

4) The voltage of an AC circuit can be written as $V = V_0 \cos(\omega t + \phi)$. Calculate the RMS voltage over the period $t = 0$ to $t = 2\pi/\omega$.

5) Evaluate

i) $\int\limits_0^\infty e^{-2x}\mathrm{dx}$.

ii) $\int\limits_1^e \ln|x|\mathrm{dx}$.

6) Determine $\langle y \rangle$ for $y = xe^{-2x}$ for x between zero and one.

8 MT1 Homework Problems: Week 8

1) Find the volume of the solid generated when the lamina $y = e^{-x}$, between $x = 0$ and $x = +\infty$ is rotated about the x axis.

2) Find the volume of the solid generated when the lamina $y = x^2 + \frac{1}{x}$, between $x = 1$ and $x = 2$ is rotated about the y axis.

3) Find the centroid $(\overline{x}, \overline{y})$ of the lamina bounded by the $x-$axis, $y = 1 + e^x$, $x = 0$ and $x = 1$.

4) Find the coordinates $(\overline{x}, \overline{y})$ of the center of mass of the solid in Question (**2**).

9 MT1 Homework Problems: Week 9

1) Evaluate

i) $\int_0^a \int_0^b xy^2 + x \mathrm{dydx}$

ii) $\int_1^a \int_0^b xy^2 + \frac{1}{x} \mathrm{dydx}$

2) By suitable integration determine the volume bound by the surfaces $z = 1 + x\sin(\pi y)$, $z = 0$, $x = 0$, $x = 1$, $y = 0$, and $y = 1$.

3) The moment of inertia of an element of arc at a perpendicular radius r from its axis of rotation is given by $\mathrm{dI} = \rho \mathrm{r}^3 \mathrm{dr} \mathrm{d}\theta$, where ρ is the mass per unit area. By suitable integration determine the moment of inertia of a disc of radius R.

4) By suitable integration obtain the volume of a sphere of radius a given that a volume element in spherical polar co-ordinates is $\mathrm{dV} = \mathrm{r}^2 \sin(\theta) \mathrm{d}\phi \mathrm{d}\theta \mathrm{dr}$.

5) By suitable integration obtain the volume of an ellipsoidal cylinder of length L, where $x = a\sin\theta$ and $y = b\cos\theta$.