PHY218 : Homework 5

1. A series electric circuit contains a resistance R, capacitance C and power source supplying a time-dependent electromotive force $V(t)$. The charge q on the capacitor obeys

$$R\frac{dq}{dt} + \frac{q}{C} = V(t)$$

Assuming that initially, at time $t = 0$, there is no charge on the capacitor, and given that $V(t) = V_0 \sin \omega t$, find the charge on the capacitor as a function of time. [10]

2. The differential equation

$$P(x, y)dx + Q(x, y)dy = 0$$

is exact. If

$$\phi(x, y) = \int_{x_0}^{x} P(x, y) + \int_{y_0}^{y} Q(x_0, y)dy$$

show that

$$\frac{\partial \phi}{\partial x} = P(x, y), \quad \frac{\partial \phi}{\partial y} = Q(x, y)$$

Hence show that $\phi(x, y) = constant$ is a solution.
Apply this to solve the equation

$$x\frac{dy}{dx} + 3x + y = 0$$

[6+2+4]

3. Show that

$$y(x) = \exp\left[-\int^{x} p(t)dt\right]\left\{\exp\left[\int^{s} p(t)dt\right] q(s)ds + C\right\}$$

is a solution of

$$\frac{dy}{dx} + p(x)y(x) = q(x)$$

by differentiating the expression for $y(x)$ and substituting in the differential equation. [10]

4. The rate of evaporation from a particular spherical drop of liquid (constant density) is proportional to its surface area. Assuming this to be the sole mechanism of mass loss, find the radius of the drop as a function of time.

Hint : The volume of a sphere of radius R is $\frac{4}{3}\pi R^3$. The surface area is $4\pi R^2$.

5. Radioactive nuclei decay according to the law

$$\frac{dN}{dt} = -\lambda N$$

where N is the concentration of a given nuclide and λ the corresponding decay constant. In a radioactive series of n nuclides, starting with N_1,

$$\begin{aligned} \frac{dN_1}{dt} &= -\lambda_1 N_1 \\ \frac{dN_2}{dt} &= \lambda_1 N_1 - \lambda_2 N_2, \text{ and so on} \end{aligned}$$

Find $N_2(t)$ for the conditions $N_1(0) = N_0$ and $N_2(0) = 0$.

[12]