PHY218 : Homework 10

1. Take the contour C to be a square with sides of length 2, centred around the origin. Do the integral $\oint_C z$ by writing it as a sum of four pieces. Comment on the relation of your answer to Cauchy's theorem. [10]

2. Find the Laurent series for the functions below, about the points specified and hence find the residue at the point

$$\frac{1}{z(z+1)} \quad , \quad z=0$$
$$\frac{\sin z}{z^4} \quad , \quad z=0$$
$$\sin\frac{1}{z} \quad , \quad z=0$$
$$\frac{e^z}{z^2-1} \quad , \quad z=1$$

[4 × 5]

3. Find the residues of the following functions at the specified points

$$\frac{1}{(3z+2)(2-z)} \text{at } z=\frac{-2}{3} \text{ and } z=2$$
$$\frac{1}{(1-2z)(5z-4)} \text{ at } z=\frac{1}{2} \text{ and } z=\frac{4}{5}$$
$$\frac{(z-2)}{z(1-z)} \text{at } z=0, z=1$$
$$\frac{e^{3z}-3z-1}{z^4} \text{ at } z=0$$

[4 × 5]

4. Use the residue theorem to calculate

$$\int_0^{2\pi} \frac{d\theta}{13+5\sin\theta}$$
$$\int_0^{2\pi} \frac{d\theta}{5-4\sin\theta}$$
$$\int_{-\infty}^{\infty} \frac{dx}{x^2+4x+5}$$
$$\int_0^{\infty} \frac{x^2}{x^4+16} dx$$

[4 × 5]