PHY218 : Homework 1

1. Calculate the Matrix product AB for

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$
$$B = \begin{pmatrix} 2 & 1 & 3 \\ 5 & 4 & 6 \\ 8 & 9 & 7 \end{pmatrix}$$

[5]

2. Use the formula

$$(AB)_{ij} = \sum_k A_{ik} B_{kj}$$

to show that

$$(AB)^T = B^T A^T$$
$$(AB)^\dagger = B^\dagger A^\dagger$$

The transpose and hermitian conjugate are defined by

$$(A^T)_{ij} = A_{ji}$$
$$(A^\dagger)_{ij} = A^*_{ji}$$

where $*$ denotes complex conjugation. [10]

3. Consider a spherical globe of radius R and a point P on its surface at latitude θ and longitude ϕ. What are the Cartesian coordinates of the displacement vector from the origin to the point P. (You may wish to review "spherical coordinates" from the reference books or earlier math courses). [10]

4. Check whether the following sets of vectors are **linearly independent** or not

$$\mathbf{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad \mathbf{w} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$\mathbf{u} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}, \mathbf{v} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \quad \mathbf{w} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$
$$\mathbf{u} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}, \mathbf{v} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \quad \mathbf{w} = \begin{pmatrix} -4 \\ 1 \\ -2 \end{pmatrix}$$

[10]

5. Prove that the complex numbers $\mathbb{C}$ form a vector space of dimension 2 over the real numbers $\mathbb{R}$. [12]

6. The following equation gives the matrix elements of the matrix product ABC

$$(ABC)_{ij} = \sum_k \sum_l A_{ik} B_{kl} C_{lj}$$

where A is an $M \times N$ matrix, B is $N \times K$ and C is a $K \times L$ matrix.

- Which of the indices in the above equation are *free indices* and which are *dummy indices* ? [4]
- More precisely, the above is a system of equations. How many equations are there in the system ? [4]
- What are the ranges of the summations over k and l ? [6]
- Derive the equation starting from $(AB)_{ij} = \sum_k A_{ik} B_{kj}$. [6]