PHY218 : Exercise 6

1. Solve the following differential equations.
(i)

$$\begin{aligned}\frac{\partial P(x,t)}{\partial x} + x &= 0\\ \frac{\partial P(x,t)}{\partial t} &= t\end{aligned}$$

(ii)

$$\begin{aligned}\frac{\partial U(x,y)}{\partial x} &= U(x,y)\\ \frac{\partial U(x,y)}{\partial y} &= yU(x,y)\end{aligned}$$

(iii)

$$\begin{aligned}\frac{\partial R(x,t)}{\partial x} &= t\\ \frac{\partial R(x,t)}{\partial t} - \frac{R(x,t)}{t} &= 0\\ \text{for the boundary condition } R(0,t) &= 3t\end{aligned}$$

[5+5+5]

2. In two dimensions, a point charge $+Q$ is placed at the origin. The electric potential $V(x,y)$ is found to satisfy :

$$\begin{aligned}\frac{\partial V(x,y)}{\partial x} &= \frac{-Qx}{x^2+y^2}\\ \frac{\partial V(x,y)}{\partial y} &= \frac{-Qy}{x^2+y^2}\end{aligned}$$

Find the electric potential as a function of the 2d coordinates. [6]

3. Given that that the Heaviside step function is defined as

$$H(x) = \begin{cases} 0 & \text{for } x < 0\\ 1 & \text{for } x > 0\\ 1/2 & \text{for } x = 0\end{cases}$$

show that

$$\delta(x) = \frac{d}{dx}H(x)$$

[5]

4. Evaluate the integrals.

(i)

$$\int_{-1}^{1} x^2 \delta(x) dx$$

(ii)

$$\int_{0}^{\pi} \sin(x) \delta(x - \frac{\pi}{4}) dx$$

(iii)

$$\int_{1}^{\infty} e^{-x^2} \delta(x) dx$$

(iv)

$$\int_{x=-\infty}^{\infty} \int_{y=-\infty}^{+\infty} \frac{x+1}{y+4} \delta(x)\delta(y) dx dy$$

(v)

$$\int_{x=-\infty}^{\infty} \int_{y=-\infty}^{\infty} \int_{z=-\infty}^{\infty} (z+1) \sin(x) \cos(y) \delta(x - \frac{\pi}{2}) \delta(y - \pi) \delta(z) dx dy dz$$

[3+3+3+3+3]