EX SHEET 7 : Mathematica for differential equations

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PHY218 : Exercise 7

In this lab assignment we will use Mathematica to solve some first and second order ordinary differential equations. We will look at some linear and non-linear examples utilising both analytic and numerical methods. Each question carries 15 points.

Exercise 1

Solve the differential equation:-

(i)

$$x\frac{dy(x)}{dx} + 3y(x) = 2$$

subject to the boundary condition $y(1) = 2$.

(ii)

$$\frac{dy(x)}{dx} = \frac{x + 2y(x) + 3}{3x + 6y(x) + 7}.$$

(iii)

$$\frac{dy(x)}{dx} + \frac{y(x)}{x} = \sin x$$

subject to the boundary condition that $y(\pi) = 0$.

Plot the solutions to part (i) and part (iii) above using the Mathematica command `Plot`. Choose a suitable range for x in each case to illustrate the shape of the solution.

> *NOTE:* If you obtain a solution containing the '`ProductLog`' function then you can rearrange the solution in terms of the constant of integration, `C[1]`, by using `Solve` and `Simplify`. Remember that this constant is arbitrary so you can then scale this solution into a more convenient form.

Exercise 2

Solve the differential equation

$$2xy(x)\frac{dy(x)}{dx} = x^2 + y(x)^2$$

subject to the boundary condition $y(1) = 2$. Then, solve this equation again but using the numerical `NDSolve` routine.

Compare the two solutions by plotting them. Numerical solutions may be plotted in Mathematica by assigning the output of `NDSolve` to a variable `s` and then substituting the command `Evaluate[y[x] /. s]` for the function in the `Plot` command.

Exercise 3

Solve the following differential equations by using the Mathematica command `DSolve`:-

(i)

$$\frac{d^2y(x)}{dx^2} - x^3 + 2x + 1 = 0. \tag{1}$$

(ii)

$$\frac{d^2x(t)}{dt^2} + a\frac{dx(t)}{dt} + x(t) = 0 \tag{2}$$

subject to the initial conditions that $x(0) = 1$ and $\frac{dx(0)}{dt} = -1$.

In order to include a differential as an initial condition we can make use of Mathematica's symbolic differential operator `x'[a]`. This represents the differential $dx(t)/dt$ at the point $t = a$.

(iii) Equation (2) represents the motion of a harmonic oscillator with a damping force described by a constant a. Using the replacement operator '`/.`', substitute different values for a into the solution found in part (ii) (e.g. $a = 0.1$ and $a = 10$). Plot the results in order to compare the affect of small and large damping.

Exercise 4

Solve the differential equation

$$\frac{d^2x(t)}{dt^2} + x(t)(x^2(t) - 1) + t = 0 \tag{3}$$

numerically, subject to the initial conditions that $x(0) = 1$ and $dx(0)/dt = 0$. Use `Plot` to plot the solution for the domain $t \in [0, 10]$.

This equation describes the motion of a particle of unit mass moving in a double well potential, with minima located at $x = 1$ and $x = -1$, under the influence of a driving force that grows with t. Initially the particle is stationary at the bottom of the well at $x = 1$, with time it is pushed into the second well and oscillates about it while continuing to move with the driving force.