PHY218 : Mathematical Techniques 3 – Exercise Class 2

1. Calculate the eigenvalues and eigenvectors of the 2×2 matrices

$$\sigma_3 \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$
$$\sigma_2 \equiv \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$
$$\sigma_1 \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

You may wish to revise your MT2 notes on this or consult your course text-book from MT2 (Stroud pages 558-564). [18]

2. The coomplex conjugate A^* of a matrix A is defined by

$$(A^*)_{ij} = (A_{ij})^*$$

In words, the matrix elements of the complex conjugated matrix are the complex conjugates of the matrix elements of the matrix.

Prove, using $(AB)_{ij} = \sum_k A_{ik} B_{kj}$, that

$$(AB)^* = A^* B^*$$

[8]

3. Calculate the determinant and inverse of each of the following matrices

$$\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} , \begin{pmatrix} 1 & 2 & 1 \\ -1 & 2 & 1 \\ 3 & 5 & 4 \end{pmatrix}$$

You may need to revise your MT2 notes to answer this question. If you don't have them handy, consult Chapter 8 of Riley, Hobson and Bence. (pages 263-266) [2+2+4+4]

4. In matrix multiplication, it is not in general true that $AB = BA$. The difference between the two is called the commutator and written as $[A, B] = AB - BA$.

Using the matrices $\sigma_1, \sigma_2, \sigma_3$ in Problem 1, calculate the three commutators $[\sigma_1, \sigma_2], [\sigma_1, \sigma_3], [\sigma_2, \sigma_3]$. [3+3+3]

Matrix multiplication is associative. Verify that $(\sigma_1\sigma_2)\sigma_3 = \sigma_1(\sigma_2\sigma_3)$. [4]