PHY218 : Mathematical Techniques 3 - Exercise Sheet 1

1. A displacement on a plane can be described by a magnitude r and a direction θ with respect to the positive x axis. Calculate the Cartesian components of this displacement vector in terms of r and θ. [5]

2. For matrices

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \quad B = \begin{pmatrix} 5 & 2 \\ 4 & 1 \end{pmatrix}$$

calculate the matrix products $AB, BA, A^T B^T, B^T A^T$. [10]

Do you spot an interesting pattern which could be general ?

3. An $m \times k$ matrix is one which has m rows and k columns. If A is an $m \times k$ matrix and B is $k \times n$, the matrix product AB is an $m \times n$ matrix.

Illustrate this in the following multiplications.

$$\begin{aligned}
A &= \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \;;\; B = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \\
A &= \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \;;\; B = \begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix} \\
A &= \begin{pmatrix} 1 & 2 \end{pmatrix} \;;\; B = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} \\
A &= \begin{pmatrix} 1 & 2 \end{pmatrix} \;;\; B = \begin{pmatrix} 1 \\ 4 \end{pmatrix} \\
A &= \begin{pmatrix} 1 \\ 2 \end{pmatrix} \;;\; B = \begin{pmatrix} 1 & 4 \end{pmatrix}
\end{aligned}$$

[10]

4. Check whether the following sets of vectors are **linearly independent** or not

$$\mathbf{v} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \quad \mathbf{w} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\mathbf{v} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \quad \mathbf{w} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$\mathbf{v} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \quad \mathbf{w} = \begin{pmatrix} -4 \\ 2 \\ -2 \end{pmatrix}$$

$$\mathbf{u} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad \mathbf{v} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \quad \mathbf{w} = \begin{pmatrix} 0 \\ 1 \\ 4 \end{pmatrix}$$

[2+4 + 4 + 5 = 15]